3维流形的映射度问题和拓扑量子场论

陈海苗

北京工商大学

摘要

给定3维定向闭流形M, N及整数k, 是否存在连续映射$f : M \to N$使$\deg f = k$? 对该问题的研究已经有近30年的历史，但仍有很多问题未解决. 当$N = S^3/\Gamma$, 其中Γ为自由作用在S^3上的有限群时, 来自拓扑量子场论的*Dijkgraaf-Witten*不变量可以给出完整的回答.

取定$[\omega] \in H^3(B\Gamma; U(1))$, M的Dijkgraaf-Witten不变量定义为

$$Z(M) = \frac{1}{\#\Gamma} \cdot \sum_{\Phi \in \hom(\pi_1(M), \Gamma)} \langle F(\Phi)^*[\omega], [M] \rangle,$$

其中$F(\Phi) : M \to B\Gamma$是诱导Φ的连续映射, 其同伦类唯一, 而$\langle -, - \rangle$是配对$H^3(M; U(1)) \times H_3(M; \mathbb{Z}) \to U(1) \subset \mathbb{C}$.

摘要

Given two oriented closed 3-manifolds M, N and an integer k, does there exist a continuous mapping $f : M \to N$ with $\deg f = k$? This problem has been studied for nearly 30 years, with still many unknowns. When $N = S^3/\Gamma$ where Γ is a finite group acting freely on S^3, a complete answer can be given by *Dijkgraaf-Witten invariant*, which arises from topological quantum field theory.

Fix $[\omega] \in H^3(B\Gamma; U(1))$, the Dijkgraaf-Witten invariant of M is

$$Z(M) = \frac{1}{\#\Gamma} \cdot \sum_{\Phi \in \hom(\pi_1(M), \Gamma)} \langle F(\Phi)^*[\omega], [M] \rangle,$$

where $F(\Phi) : M \to B\Gamma$ is a mapping inducing Φ which is unique up to homotopy, and $\langle -, - \rangle$ is the paring $H^3(M; U(1)) \times H_3(M; \mathbb{Z}) \to U(1) \subset \mathbb{C}$.